

LAST NAME: \_\_\_\_\_

ASSIGNED CLASS NUMBER \_\_\_\_\_

## Forces, Mechanical Work and Energy

1 A small container of water is placed on a carousel inside a microwave oven, at a radius of 12.0 cm from the center. The turntable rotates steadily, turning through one revolution in each 7.25 s. What angle does the water surface make with the horizontal?

The container moves at speed  $v = \frac{2\pi r}{T} = \frac{2\pi(0.12 \text{ m})}{7.25 \text{ s}} = 0.104 \text{ m/s}$ .

The top layer of water feels a downward force of gravity  $mg$  and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104 \text{ m/s})^2}{0.12 \text{ m}} = m 9.01 \times 10^{-2} \text{ m/s}^2.$$

It behaves as if it were stationary in a gravity field pointing downward and outward  $\tan^{-1} \frac{0.0901 \text{ m/s}^2}{9.8 \text{ m/s}^2} = \boxed{0.527^\circ}$ .

Its surface slopes upward toward the outside, making this angle with the horizontal.

2A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by  $a = g - bv$ . After falling 0.500 m, the Styrofoam effectively reaches terminal speed, and then takes 5.00 s more to reach the ground. (a) What is the value of the constant  $b$ ? (b) What is the acceleration at  $t = 0$ ? (c) What is the acceleration when the speed is 0.150 m/s?

$a = g - bv$  When  $v = v_T$ ,  $a = 0$  and  $g = bv_T$   $b = \frac{g}{v_T}$  The Styrofoam falls 1.50 m at constant speed  $v_T$  in

5.00 s. Thus,  $v_T = \frac{y}{t} = \frac{1.50 \text{ m}}{5.00 \text{ s}} = 0.300 \text{ m/s}$  Then  $b = \frac{9.80 \text{ m/s}^2}{0.300 \text{ m/s}} = \boxed{32.7 \text{ s}^{-1}}$

(b) At  $t = 0$ ,  $v = 0$  and  $a = g = \boxed{9.80 \text{ m/s}^2}$  down

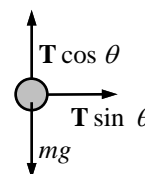
(c) When  $v = 0.150 \text{ m/s}$ ,  $a = g - bv = 9.80 \text{ m/s}^2 - (32.7 \text{ s}^{-1})(0.150 \text{ m/s}) = \boxed{4.90 \text{ m/s}^2}$  down

3A 0.500-kg object is suspended from the ceiling of an accelerating boxcar as in Figure 6.13. If  $a = 3.00 \text{ m/s}^2$ , find (a) the angle that the string makes with the vertical and (b) the tension in the string.

The only forces acting on the suspended object are the force of gravity  $mg$  and the force of tension  $T$ , as shown in the free-body diagram. Applying Newton's second law in the  $x$  and  $y$  directions,

$$\sum F_x = T \sin \theta = ma \quad (1)$$

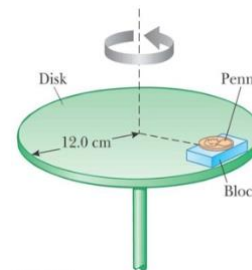
$$\sum F_y = T \cos \theta - mg = 0 \text{ or } T \cos \theta = mg \quad (2)$$



(a) Dividing equation (1) by (2) gives  $\tan \theta = \frac{a}{g} = \frac{3.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.306$ . Solving for  $\theta$ ,  $\theta = \boxed{17.0^\circ}$

(b) From Equation (1),  $T = \frac{ma}{\sin \theta} = \frac{(0.500 \text{ kg})(3.00 \text{ m/s}^2)}{\sin(17.0^\circ)} = \boxed{5.12 \text{ N}}$ .

4 A penny of mass 3.10 g rests on a small 20.0-g block supported by a spinning disk. The coefficients of friction between block and disk are 0.750 (static) and 0.640 (kinetic) while those for the penny and block are 0.520 (static) and 0.450 (kinetic). What is the maximum rate of rotation in revolutions per minute that the disk can have, without the block or penny sliding on the disk?



Use opposite side of this page for your diagrams and answers

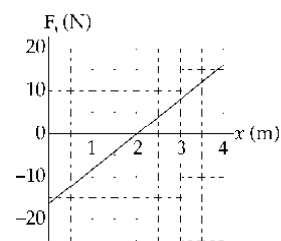
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5 The force acting on a particle is  $F_x = (8x - 16) \text{ N}$ , where  $x$  is in meters. (a) Make a plot of this force versus  $x$  from  $x = 0$  to  $x = 3.00 \text{ m}$ . (b) From your graph, find the net work done by this force on the particle as it moves from  $x = 0$  to  $x = 3.00 \text{ m}$ .

a)  $F_x = (8x - 16) \text{ N}$  See figure to the right

$$\text{b) } W_{\text{net}} = \frac{-(2.00 \text{ m})(16.0 \text{ N})}{2} + \frac{(1.00 \text{ m})(8.00 \text{ N})}{2} = \boxed{-12.0 \text{ J}}$$



6 A baseball outfielder throws a  $0.150\text{-kg}$  baseball at a speed of  $40.0 \text{ m/s}$  and an initial angle of  $30.0^\circ$ . What is the kinetic energy of the baseball at the highest point of its trajectory?

$$\text{At start, } \mathbf{v} = (40.0 \text{ m/s}) \cos 30.0^\circ \hat{\mathbf{i}} + (40.0 \text{ m/s}) \sin 30.0^\circ \hat{\mathbf{j}}$$

$$\text{At apex, } \mathbf{v} = (40.0 \text{ m/s}) \cos 30.0^\circ \hat{\mathbf{i}} + 0 \hat{\mathbf{j}} = (34.6 \text{ m/s}) \hat{\mathbf{i}}$$

$$\text{And } K = \frac{1}{2} m v^2 = \frac{1}{2} (0.150 \text{ kg}) (34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$$

7 A  $40.0\text{-kg}$  box initially at rest is pushed  $5.00 \text{ m}$  along a rough, horizontal floor with a constant applied horizontal force of  $130 \text{ N}$ . If the coefficient of friction between box and floor is  $0.300$ , find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

$$\sum F_y = m a_y: \quad n - 392 \text{ N} = 0 \quad n = 392 \text{ N} \\ f_k = \mu_k n = (0.300)(392 \text{ N}) = 118 \text{ N}$$

$$\text{(a) } W_F = F \Delta r \cos \theta = (130)(5.00) \cos 0^\circ = \boxed{650 \text{ J}}$$

$$\text{(b) } \Delta E_{\text{int}} = f_k \Delta x = (118)(5.00) = \boxed{588 \text{ J}}$$

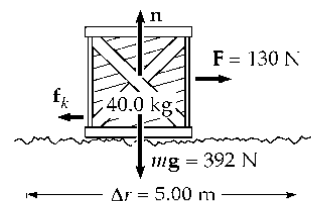
$$\text{(c) } W_n = n \Delta r \cos \theta = (392)(5.00) \cos 90^\circ = \boxed{0}$$

$$\text{(d) } W_g = m g \Delta r \cos \theta = (392)(5.00) \cos(-90^\circ) = \boxed{0}$$

$$\text{(e) } \Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$$

$$\frac{1}{2} m v_f^2 - 0 = 650 \text{ J} - 588 \text{ J} + 0 + 0 = \boxed{62.0 \text{ J}}$$

$$\text{(f) } v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0 \text{ J})}{40.0 \text{ kg}}} = \boxed{1.76 \text{ m/s}}$$



8  $0.1\text{kg}$  Particle moves under influence of conservative force whose potential energy is shown in the diagram. At  $t=0$  particle has  $K=4\text{J}$  at  $x=8\text{m}$ .

a) What are the turning points for this particle?

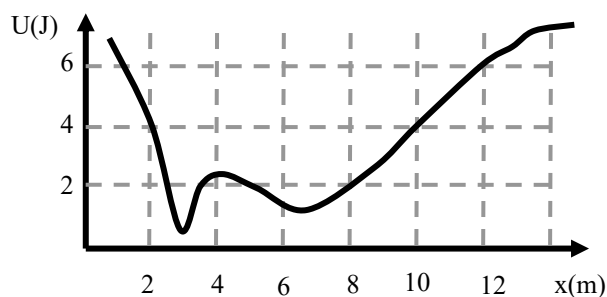
b) what is the particle  $v$  at  $x=2\text{m}$ ?c) what is the orientation of the force acting on it at  $x=8\text{m}$ ?

$$E_{\text{tot}} = 2\text{J} + 4\text{J} = 6\text{J}$$

$$\text{a) } x_L = 1\text{m} \quad x_R = 12\text{m}$$

$$\text{b) } K(2\text{m}) = 2\text{J} \text{ so } v(2\text{m}) = 6.32\text{m/s}$$

$$\text{c) } \text{slope } dU/dx > 0 \text{ so } F_x < 0$$



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4 For the block to remain stationary,  $\sum F_y = 0$  and  $\sum F_x = m a_x$ .

$$n_1 = (m_p + m_b)g \text{ so } f \leq \mu_{sl} n_1 = \mu_{sl} (m_p + m_b)g.$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore (m_p + m_b) \frac{v_{max}^2}{r} = \mu_{sl} (m_p + m_b)g$$

$$\text{or } v_{max} = \sqrt{\mu_{sl} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}.$$

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

$$\text{and } \sum F_x = m a_x \Rightarrow f_p = m_p \frac{v^2}{r}.$$

When the penny is about to slip on the block,  $f_p = f_{p, max} = \mu_{sl} n_2$

$$\text{or } \mu_{sl} m_p g = m_p \frac{v_{max}^2}{r}$$

$$v_{max} = \sqrt{\mu_{sl} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{max}}{2\pi r} = (0.782 \text{ m/s}) \left[ \frac{1 \text{ rev}}{2\pi(0.120 \text{ m})} \right] \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{62.2 \text{ rev/min}}.$$

